Ising Dynamics with Damping

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We show for the Ising model that is possible construct a discrete time stochastic model analogous to the Langevin equation that incorporates an arbitrary amount of damping. It is shown to give the correct equilibrium statistics and is then used to investigate nonequilibrium phenomena, in particular, magnetic avalanches. The value of damping can greatly alter the shape of hysteresis loops, and for small damping and high disorder, the morphology of large avalanches can be drastically effected. Small damping also alters the size distribution of avalanches at criticality.

I. INTRODUCTION

In many situations, it is useful to discretize continuous degrees of freedom to better understand them, both from a theoretical standpoint and for numerical efficiency. Ising models are perhaps the best example of this and have been the subject of numerous theoretical and numerical studies. Renormalization group arguments[1] have explained the reason why this discretization gives equilibrium critical properties of many experimental systems, and these kinds of arguments have been extended to understanding their equilibrium dynamics[2]. For nonequilibrium situations, such as the study of avalanches, such arguments probably do also apply to large enough length and time scales as well. However there are many situations where it would be desirable to understand smaller length scales where other factors should become relevant.

This is particularly true with dynamics of magnetic systems, where damping is often weak in comparison to precessional effects. For studies of smaller scales, it has been necessary to use more time consuming micromagnetic simulations utilizing continuous degrees of freedom, such as the Landau Lifshitz Gilbert equations[3] which is a kind of Langevin equation that gives the stochastic evolution of Heisenberg spins.

$$\frac{d\mathbf{s}}{dt} = -\mathbf{s} \times (\mathbf{B} - \gamma \mathbf{s} \times \mathbf{B}), \qquad (1)$$

where $\mathbf{s}$ is a microscopic magnetic moment, $\mathbf{B}$ is the local effective field, and γ is a damping factor, measuring the relative importance of damping to precession. In real materials it ranges[4] from small damping $\gamma = .01$, to 1. In contrast, the dynamical rules implemented for Ising models are most often "relaxational" so that energy is instantaneously dissipated when a spin flips, as with the Metropolis algorithm.

However there is a class of "microcanonical" Ising dynamics[5] reviewed in section II where auxiliary degrees of freedom are introduced and all moves conserve the total energy. The other degrees of freedom can be taken to be variables associated with each spin, and allowed moves can change both the state of the spins and the auxiliary variables. This can be thought of crudely, as a discretized analogy to molecular dynamics, and is also similar to discrete lattice gas models of fluids[6,7]. These models give the correct equilibrium Ising statistics of large systems and can also be used to understand dynamics in a different limit than the relaxational case.

Real spin systems are intermediate between these two kinds of dynamics and as mentioned above, are better described by Langevin dynamics. In the context of spins, the question posed and answered here is: how does one formulate a discrete time version of stochastic dynamics that includes damping and gives the correct equilibrium statistics? In section II A we are able to show that there is a fairly simple method for doing this using a combination of microcanonical dynamics, and an elegant procedure that incorporates damping and thermal noise. This procedure differs from that of the Langevin equation in that it requires non-Gaussian noise. Despite this, the noise has surprisingly simple but unusual statistics.

We will then show that this procedure gives the correct equilibrium statistics and verify this numerical in section II B with a simulation of the two dimensional Ising model with different amounts of damping.

Because the value of damping is an important physical parameter in many situations it is important that there is a straightforward way of incorporating its effects in Ising simulations. This is particularly noteworthy as Ising kinetics are a frequently used means of understanding dynamics in many condensed matter systems.

After this in section III we will turn to nonequilibrium problems where, using this approach, we can study the effects of damping on a number of interesting properties of systems displaying avalanches and Barkhausen noise[8]. We first show how to modify the kinetics for this case and then study systems in two and three dimensions. With modest amounts of computer time, we can analyze problems that are out of the reach of micromagnetic simulations and allow us to probe the effects of damping on the properties of avalanches. This is related to recent work[9]

by the present authors using both the Landau Lifshitz Gilbert equation, Eqn. 1, and theoretical approaches, to understand how relaxational dynamics of avalanches[10], are modified at small to intermediate scales by this more realistic approach. With the present approach we find new features and modifications of avalanche dynamics. We find that the shape of hysteresis loops can be strongly influenced by the amount of damping. One of the most striking findings is that there exists a parameter regime of high disorder and small damping where single system-size avalanches occur that are made up of a large number of disconnected pieces. We can also analyze the critical properties of avalanches when damping is small and give evidence that there is a crossover length scale, below which avalanches have different critical properties.

II. NON-RELAXATIONAL DYNAMICS

We start by considering a model for a magnet with continuous degrees of freedom, such as a Heisenberg model with anisotropy. The Ising approximation simplifies the state of each spin to either up or down, that is $s_i = \pm 1$, $i = 1, \ldots, N$. One important effect that is ignored by this approximation is that of spin waves that allow the transfer of energy between neighbors, and for small oscillations, give an energy contribution per spin equal to the temperature T (here we set $k_B = 1$). This motivates the idea that there are extra degrees of freedom associated with every spin that can carry (a positive) energy e_i. Creutz introduced such degrees of freedom[5] and posited that they could take any number of discrete values. He used these auxiliary variables e_i to construct a cellular automota to give the correct equilibrium statistics for the Ising model, in a very efficient way that did not require the generation of random numbers. Thus we have a Hamiltonian H_{tot} that is the sum of both spin H_{spin} and auxiliary degrees of freedom H_e: $H_{tot} = H_{spin} + H_e$. H_{spin} can be a general Ising spin Hamiltonian and $H_e = \sum_i e_i$. In our model there is a single auxiliary variable e_i associated with each lattice site i, that can take on any real value ≥ 0.

However for the purposes of trying to model dynamics of spins, it also makes sense to allow the e_i's to interact and exchange energy between neighbors. For example, one precessing spin should excite motion in its neighbors. This exchange was formulated in the context of solidification using a Potts model instead of an Ising model by Conti et $al.$[11], but can equally well be used here.

Now we can formulate a microcanonical algorithm for the Ising model using a procedure very similar to their prescription. In each step:

1. We choose a site i at random.

2. We randomly pick with equal probability either a spin or an auxiliary degree of freedom, s_i of e_i:

(a) s_i's: We attempt to move spins (such as the flipping of a single spin). If the energy cost in doing this is $\leq e_i$ we perform the move and decrease e_i accordingly. Otherwise we reject the move.

(b) e_i's: We pick a nearest neighbor j, and repartition the total energy with uniform probability between these two variables. That is, after repartitioning, $e'_i = (e_i + e_j)r$ and $e'_j = (e_i + e_j)(1 - r)$, where $0 < r < 1$ is uniform random variable.

Note that these rules preserve the total energy and the transitions between any two states have the same probability. Therefore this will give the correct microcanonical distribution. For large N, this is, for most purposes[12], equivalent to the canonical distribution $\propto \exp{(-\beta H_{tot})}$. Note that the probability distribution for each variable e_i, $P(e_i) = \beta \exp{(-\beta e_i)}$, so that the $\langle e_i \rangle = T$. That is, measurement of average of e_i's directly gives the effective temperature of the system.

A. Extension To Damping

The question we asked, is how to extend this equilibrium simulation method to include damping. In this case the system is no longer closed and energy is exchanged with an outside heat bath through interaction with the auxiliary variables. As with the Langevin equation, there are two effects. The first is that the energy is damped. Call the dissipation parameter for each step α, which will lie between 0 and 1. Then at each time step we lower the energy with $e_i \to \alpha e_i$ for all sites i. By itself, this clearly will not give a system at finite temperature and we must also include the second effect of a heat bath, which adds energy randomly to the system. In the case of the Langevin equation, a Gaussian noise term $n(t)$ is added to keep the system at finite temperature. A discretized version of this, that evolves the energy $e(t)$ at time step t is

$$e(t + 1) = \alpha e(t) + n(t). \tag{2}$$

This equation will not work if the noise $n(t)$ is Gaussian as this does not give the Gibbs distribution $P_{eq}(e) = \beta \exp{(-\beta e)}$. Therefore we need to modify the statistics of $n(t)$. It is possible to do so if we choose $n(t)$ at each time t from a distribution

$$p(n) = \alpha \delta(n) + (1 - \alpha)\beta e^{-\beta n}\theta(n) \tag{3}$$

where θ is the Heaviside step function. To show this, we write down the corresponding equation for the evolution of the probability distribution for e:

$$P(e', t+1) = \langle \delta(e' - (\alpha e + n)) \rangle = \int \int P(e,t)p(n)\delta(e' - (\alpha e + n))de\ dn \qquad (4)$$

We require that the P as $t \to \infty$ obeys $P(e, t+1) = P(e,t) = P_{eq}(e) = \beta \exp(-\beta e)$, for $e > 0$. It is easily verified that by choosing this form of $P(e,t)$ and by choosing $P(n)$ as in Eq. 3, we satisfy Eq. 4.

Therefore to add damping to this model, we add the following procedure to the steps stated above:

3. Choose a uniform random number $0 < r < 1$. If $r < \alpha$, then $e_i \to \alpha e_i$. Otherwise $e_i \to \alpha e_i - T \ln(r')$, where r' is another uniform random number between 0 and 1.

If we assume that the probability distribution for the total system is of the form $P_{Gibbs} \propto \exp(-\beta H_{tot}) = \exp(-\beta H_{spin}) \exp(-\beta H_e)$, we will now show that the steps 1, 2, and 3, of this algorithm preserve this distribution. Following the same reasoning as above for the microcanonical simulation, moves implementing steps 1 and 2 do not change the total energy, and they preserve the form of P_{Gibbs} because P_{Gibbs} depends only on the total energy (H_{tot}), and 1 and 2 explore each state in an energy shell with uniform probability. Because of the form of P_{Gibbs}, its dependence on the variable e_i is $\propto \exp(-\beta e_i)$. According to the above argument, after step 3, it will remain unchanged. Therefore all steps in this algorithm leave P_{Gibbs} unchanged. The algorithm is also ergodic, and therefore this will converge to the Gibbs distribution[13] as $t \to \infty$.

Because the steps each preserve the Gibbs distribution, the ordering of them is not important in preserving equilibrium statistics. For example, we could sweep through the lattice sequentially instead of picking i at random. We could perform step 3 after steps 1 and 2 were performed N times.

B. Equilibrium Tests

We performed tests on this algorithm and verified that it did indeed work as expected. We simulated the two dimensional Ising model on a 128^2 lattice with different values of the damping parameter, and compared it with the exact results. The average magnetization per spin m is plotted in Fig. 1 as a function of the temperature T and compared with the exact result[14] for large N (dashed curve). The $\times$'s are the case $\alpha = 1$, which is then just an implementation of the microcanonical method[11] described above. In this case, the temperature was obtained by measuring $\langle e_i \rangle$ because the energy was fixed at the start of the simulation. The only point which is slightly off the exact solution is in the critical region, as is to be expected. The case $\alpha = 0.5$ is shown with the $+$'s and lie on the same curve. Results were obtained for

$\alpha = 0.9$ but are so close as to be indistinguishable and are therefore not shown. We also checked that the distribution of auxiliary variables had the correct form. The probability distribution for the energy e is shown in Fig. 2. Fig. 2 plots the distribution $P(e_i)$ versus energy e_i, averaged over all sites i on a linear-log scale for $\alpha = 0.5$ and $T = 0.8$, and 1.1. The curves are straight lines over four decades and show the correct slopes, for $T = 0.8$, $\langle e_i \rangle = 0.8002$ and for $T = 1.1$, $\langle e_i \rangle = 1.1003$.
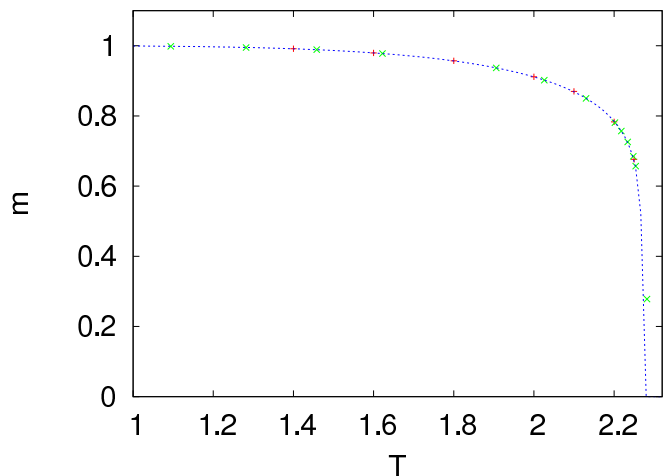

FIG. 1: Plot of results obtained for the two dimensional Ising model on a 128^2 lattice for two different values of the damping parameter. This is a plot of the average magnetization per spin m vs. T. The $\times$'s are for no dissipation, $\alpha = 1$, which is a purely microcanonical simulation. The $+$'s are for $\alpha = 0.5$. The dashed curve is the exact solution to this model in the thermodynamic limit.

III. AVALANCHE DYNAMICS

Avalanche dynamics of spin systems have been mainly studied using models that are purely relaxational. There is a whole range of interesting phenomena that have been elucidated by such studies and have yielded very interesting properties. The simplest model that can be used in this context is the random field Ising model (RFIM) with a Hamiltonian

$$\mathcal{H} = -\sum_{<ij>} J s_i s_j - \sum_i h_i s_i - h \sum s_i \qquad (5)$$

where J is the strength of the nearest neighbor coupling, h_i is a random field, with zero mean, and h is an externally applied field. A magnet is placed in a high field h
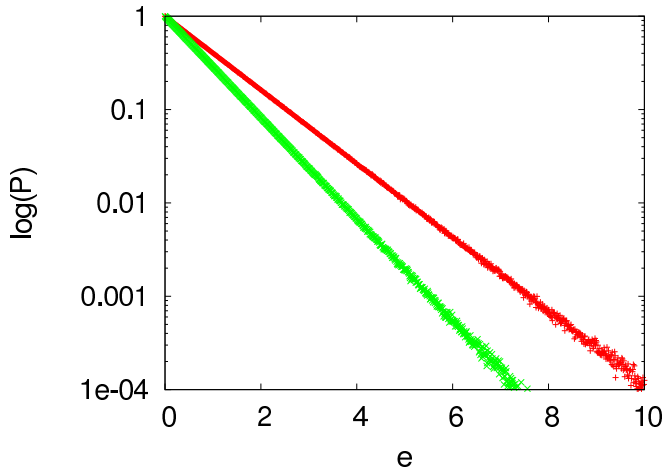
FIG. 2: Plot of results obtained for the two dimensional Ising model on a 128^2 lattice for the probability distribution for the auxiliary variables e_i, at two different temperatures with a damping parameter $\alpha = 0.5$. The upper curve is for $T = 1.1$ and the lower for $T = 0.8$.

and then this is very slowly lowered. As this happens, the spins will adjust to the new field by flipping to lower their energy. In the usual situation, the system is taken to be at $T = 0$, so that only moves that lower the energy are accepted. The flipping of one spin can cause a cascade of additional spins to flip, causing the total magnetization M to further decrease. The occurrence of these cascades is called an "avalanche". At zero temperature there is one parameter j that characterizes the system, the ratio of nearest neighbor coupling to the distribution width of the random field. One considers the behavior of a system when its starts in a high field and is slowly lowered. When j is small the system is strongly pinned and the system will have a number of small avalanches generating a smooth hysteresis loop. For large j, the system will have a system-size avalanche involving most of the spins in the system, leading to a precipitous drop in the hysteresis loop. There is a critical value of j where the distribution of avalanche sizes is a power law and self-similar scaling behavior is observed.

Here we investigate how this is modified by adding damping to these zero temperature dynamics according to the following rules:

1. The field is slowly lowered by finding the next field where a spin can flip.

2. The spins then flip, exchanging energy with auxiliary variables e_i as described above. The number of times this is attempted is n_m times the total number of spins in the system. Here we set $n_m = 16$. In more detail:

   (i) **Spin moves:** An attempt to move each spin on the lattice is performed by attempting to flip sequentially every third spin, in order to

minimize artifacts in the dynamics due to updating contiguous spins. (The lattice sites are linearly ordered using "skew" boundary conditions). Then all three sublattices are cycled over.

   (ii) **Energy moves:** Exchange of energy with nearest neighbors is performed cycling through all directions of nearest neighbors. Using the same sequence of updates, the e_i's exchange energy with their nearest neighbors in one particular direction.

   (iii) **Dissipation:** The energy of each e_i is lowered to αe_i.

3. We check for when the spins have settled down as follows: if the e_i's are not all below some energy threshold e_{thresh}, set below to be 10^{-4}, or the spin configuration has changed, step 2 is repeated until these conditions are both met.

4. When the spins have settled down, we go to step 1.

The parameters n_m and e_{thresh} were varied to check that the correct dynamics were obtained. The larger α, the smaller the dissipation and the larger the number of iterations necessary to achieve the final static configuration.
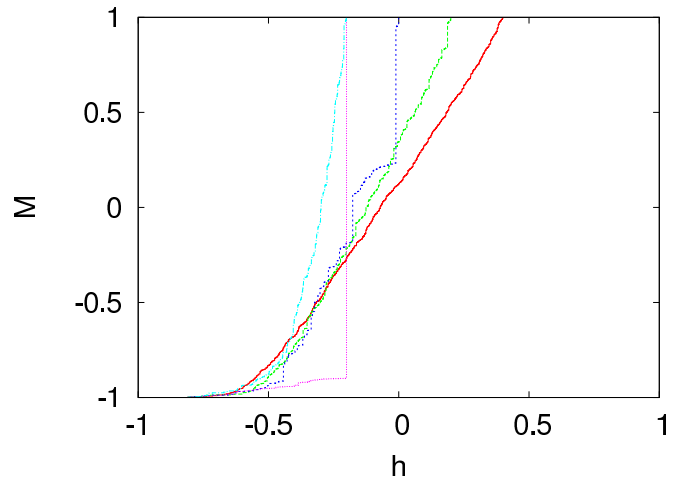

FIG. 3: The major branch of the descending hysteresis loop for 64^2 systems using different values of the damping parameter and the spin coupling. Strong damping, $\alpha = 0.5$ is shown in the left most curve (as judged from the top of the plot) for coupling $j = 0.3$ which starts decreasing from $M = 1$ at $h = -0.2$, and does not have large abrupt changes. All the other curves are for weak damping, $\alpha = 0.99$. In this case but also for $j = 0.3$, we see that although M starts to decrease at the same location as for strong damping, it drops abruptly as the field is lowered. As the coupling j is decreased, smooth curves are eventually seen again. Going left to right, as judged from the top, are $j = 0.3$, 0.25, 0.2, and 0.15.

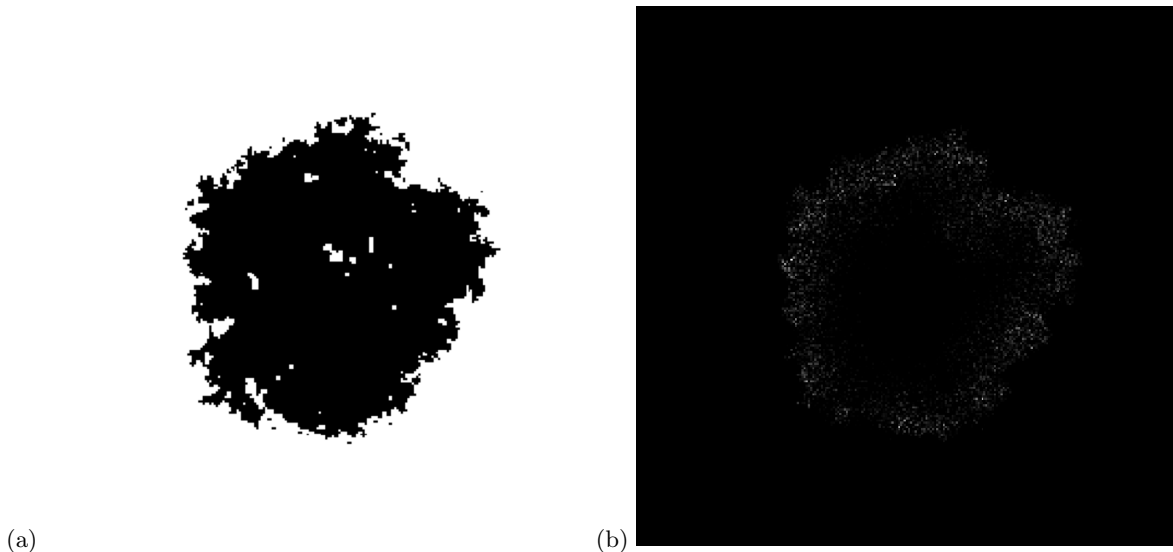(a)   (b)

FIG. 4: (a) The spin configuration for a 256^2 system with $j = 0.35$, $\alpha = 0.9$ during a system size avalanche at the field $h = -0.400007$. (b) A gray-scale plot of the auxiliary variables at the same time.
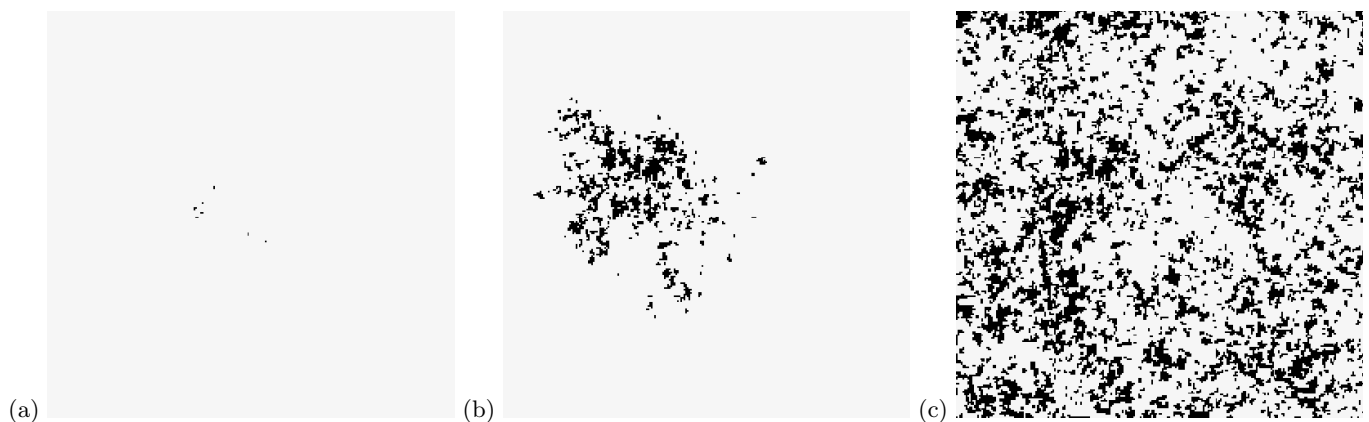

(a)   (b)   (c)

FIG. 5: Spin configurations for a 256^2 system with $j = 0.25$, $\alpha = 0.99$ during an avalanche at the field $h = -7 \times 10^{-5}$. (a) The beginning of the avalanche. (b) When the avalanche is of order of half the system size. (c) The final configuration of the avalanche.

A. Two Dimensional Patterns

We first investigate the case of two dimensions where it is simpler to visualize the avalanches in various conditions than in three dimensions. Much experimental work and theoretical work on avalanches has been done on two dimensional magnetic films and this case should be highly relevant[10].

We first examine how the hysteresis loops change as a function of the coupling j and the damping parameter α for a 64^2 system. The major downwards hysteresis loops are shown in Fig. 3 for a variety of parameters described below. We first examine strong damping $\alpha = 0.5$. For $j = 0.3$ the hysteresis curve is quite smooth with all avalanches much less than the system size (left most curve). Now consider the same value of j but with with small damping, $\alpha = 0.99$. The curve now is a single downwards step with a small tail at negative h. The lower damping has allowed that system to form a system size avalanche. The difference is due to the fact that with small damping, the energy of avalanched spins is not immediately dissipated and as a consequence, heats up neighboring spins, allowing them to more easily avalanche as well. Therefore a system size avalanche is seen in the small damping case, leading to the precipitous drop in the hysteresis loop.

When the value of the coupling j is lowered to 0.15 for $\alpha = 0.99$, smooth loops are obtained. The Fig. 3 shows intermediate values of the coupling parameter as well.

To better understand the reason why the energy of

the auxiliary variables can trigger further spins to flip, in Fig. 4 we show the state of a system during a system size avalanche for $j = 0.35$ and a moderately small damping value, $\alpha = 0.9$, with $h = -0.400007$. Fig. 4(a) shows that the flipped spins form a fairly compact cluster and Fig. 4(b) shows the corresponding values of the e_i's in a gray scale plot, suitably normalized. It has the appearance of a halo around the growth front of the avalanche. The spins in the growth front have just flipped and so energy there has not had a chance to diffuse or dissipate and so has a higher spin temperature. The interior is cold because damping has removed energy from the auxiliary degrees of freedom. This higher temperature diffuses into the the unflipped region allowing spins to flip by thermal activation.

Because large avalanches are possible for small damping in a parameter range where the relative effect of the random field is much larger, it is of interest to see if avalanches have a different morphology than typical large avalanches for high damping systems. Fig. 5 shows such spin configurations first at the beginning of the avalanche and further along during propagation when it has reached roughly half the system size, and finally when it has reached its final configuration and the maximum auxiliary variable value is $< 4 \times 10^{-4}$. The morphology of this is very different than what is seen for large avalanches with stronger coupling, for example Fig. 4. At very small fields, in this figure $h = -7 \times 10^{-5}$, surface tension precludes the formation of minority domains, but because disorder is large, there will be many small regions where the local field is much stronger and these will want to form downward oriented (black) domains. There is a finite activation barrier to forming these that can only be overcome at finite temperature. However the majority of the spins still strongly disfavor flipping. But because damping is small, heat has a chance to diffuse through these regions into the favorable regions, allowing disconnected regions to change orientation by thermal activation. Note that we have checked numerically that small damping with strong coupling also leads to compact configurations, so disorder is an essential ingredient in this new morphology.

B.   Three Dimensions

We first check that as with two dimensions, the value of the damping parameter can have a large effect on the shape of a hysteresis loop. Fig. 6 shows the downward branches of the major hysteresis loop when the only parameter that is changed is the damping, α. The system is a 32^3 lattice with $j = 0.19$. A value for high damping, $\alpha = 0.5$, is the upper line. The lower line is for small damping with $\alpha = 0.99$.

A more subtle effect, is that of damping on what happens near criticality. In this case the value of the critical j will depend on the value of α as is apparent from the results of Fig. 6. At this point, the distribution of
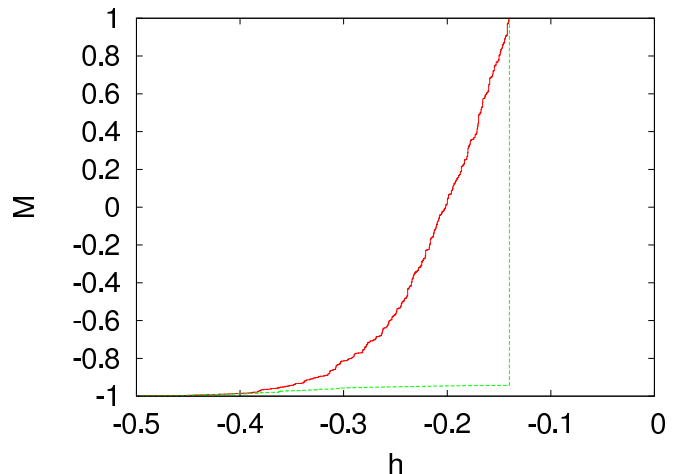

FIG. 6: The major branch of the descending hysteresis loops in two 32^3 systems with $j = 0.19$, for two different values of the dissipation, upper curve: $\alpha = 0.5$, lower curve: $\alpha = 0.99$.

avalanche sizes is expected to follow a power law distribution for large sizes. We located this point and examined system properties in this vicinity. Fig. 7 shows examples of such runs for 32^3 systems. Fig. 7(a) shows a plot of the magnetization per spin M, versus the applied field h for $j = 0.165$ and $j = 0.167$. For larger values of j, the avalanches rapidly become much larger as is seen in Fig. 6, and for smaller values, avalanches all become small. Fig. 7(b) shows a plot of the same quantity with relaxational dynamics near criticality. The avalanches take place over a much smaller range in applied field.

To quantify this difference, we studied the avalanche size distribution exponent that is obtained by calculating the distribution of avalanche sizes over the entire hysteresis loop. This was studied by averaging avalanches of many runs, (200 for $\alpha = 0.99$) for 32^3 systems and for different values of parameters. We show a comparison of the avalanche size distribution for $\alpha = 0.99$, shown with +'s and for $\alpha = 0.9$, shown with ×'s in Fig. 8. For $\alpha = 0.99$ the curve fits quite well to a power law with an exponent of $-1.4 \pm .1$ as shown in the figure. For purely relaxational dynamics, the same exponent has been carefully measured[15] to be $2.03 \pm .03$ (which is consistent with our results for relaxational dynamics on much smaller systems than theirs). With smaller damping we expect to have a crossover length corresponding to the length scale associated with the damping time, above which the dynamics should appear relaxational. $\alpha = 0.9$ appears to show such a crossover from a slope of approximately -1.4 for small avalanches, to a higher slope for large ones. A line with slope of -2 is shown for comparison and appears to be consistent with this interpretation.
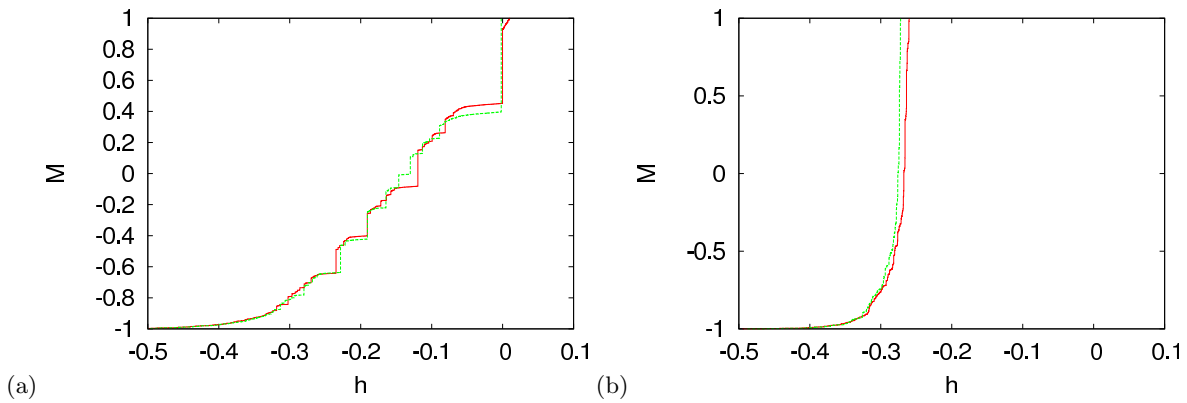
FIG. 7: (a) Magnetization versus field for the Ising model with damping described in the text. The system size is 32^3 and the two lines represent two runs close to criticality, one with a coupling of $j = 0.165$ and the other of 0.167. (b) The plot for relaxational dynamics (large damping) with couplings of .21 and .212.
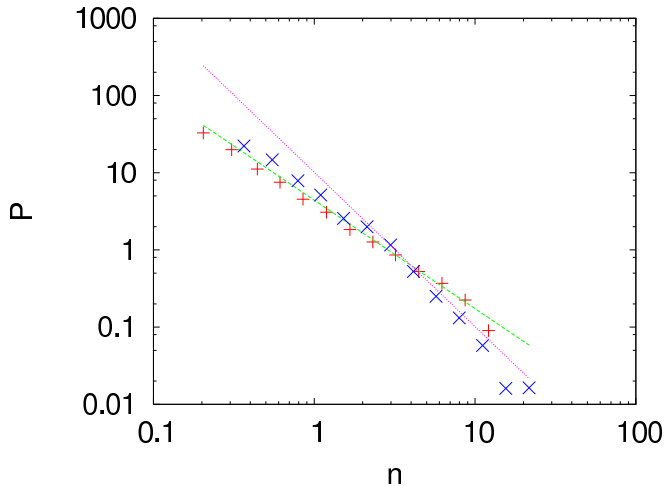

FIG. 8: The avalanche size distribution, measured of the entire hysteresis loop for $\alpha = 0.99$ (+ symbols) and $\alpha = 0.9$ ($\times$ symbols). The x-axis is the number of avalanches normalized by it's mean size. The y-axis is the normalized distribution of sizes. The less negative sloped straight line is a fit of the $\alpha = 0.99$ curve and has a slope of -1.4. The more strongly sloped one has a slope of -2.

IV. DISCUSSION

This paper has introduced a new set of dynamics for Ising models that incorporates damping in a way that has not before been achieved. The dynamics that have been devised have a lot in common with Langevin dynamics, except they are for discrete rather than continuous systems. In Langevin equations, a continuous set of stochastic differential equations are used to model a system. It differs from molecular dynamics in that thermal noise and damping are both added so that the system obeys the correct equilibrium statistics. In the case studied here, we start by considering microcanonical dy-

namics[5,11] which introduces auxiliary degrees of freedom. We then add damping and thermal noise. Whereas the thermal noise is typically Gaussian in the case of the Langevin equation, here it must be taken to be of a special exponential form, Eq. 3, in order for it to satisfy the correct equilibrium statistics.

The form of this noise, although quite unusual, can be understood, to some extent qualitatively. For large damping, or small α, the strength of the δ function becomes small, and the effect is dominated by the second term which is $\propto \exp(-\beta n)$ (for positive n). Although this is non-Gaussian, n can be thought of as a random amount of positive energy. In the Langevin equation, noise is often added to a velocity degree of freedom. In terms of a velocity, the exponential form that we have obtained would correspond to a Gaussian if this was expressed in terms of a velocity instead. In the limit of small damping, where α is close to 1, the effect of the noise becomes small because the first term, which is to add no noise, will dominate the distribution. This is in accord with what happens in the Langevin equation where if dissipation is small, little thermal noise is needed to keep the system at a given temperature.

The fact that it is possible to model damped systems in this discrete manner should have many useful applications, and is easily extended to other kinds of systems, aside from Ising models, especially in applications where computational efficiency is an important criterion.

The case of avalanches in magnetic systems is an interesting nonequilibrium use of these dynamics. Although one might expect that in most situations, for large enough distance and time scales, finite damping will be unimportant, physics at smaller scales is still of great interest and effects at those scales can propagate to larger scales. Because damping in real materials can be quite small, their effects are readily observable experimentally. This work is expected to be important at intermediate scales. We have investigated the phenomenon seen in this model with varying degrees of damping and found that

it makes a qualitative difference to many of the features seen on small and intermediate scales. This work is by no means exhaustive and there are many other effects that can be investigated by straightforward extensions. The effect of dipolar interactions in conjunction with damping could also be explored. We have chosen to update the spin and auxiliary variables at equal frequencies. Varying this should lead to a different value for the heat diffusion coefficient which should change the quantitative values for length and time scales.

The phenomena we have found was in qualitative agreement with earlier work using the Landau Lifshitz Gibbs equations[9]. As avalanches progress, the effective temperature, which we have seen can be quantified by $\langle e_i \rangle$ at site i, will increase as energy is released. This energy then diffuses to the surrounding regions, giving those spins the opportunity to lower their energy by thermal activation. This allows avalanches to more easily progress when the damping is small in contrast to relaxational dynamics, which has effectively infinite damping, $\alpha = 0$. This can lead to some substantial differences in avalanche morphology, particularly as for small damping, highly disordered systems can avalanche. At low fields this leads to a single avalanche being composed of many disconnected pieces. Experiments have been devised[16] that are close experimental realization of the two dimensional random field Ising model, and it would be interesting to determine if systems such as this one, or similar to it, show avalanches with this morphology.

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